



UNIVERSITY OF NORTH BENGAL
B.Sc. Honours 4th Semester Examination, 2023

CC8-PHYSICS
MATHEMATICAL METHODS-III

Time Allotted: 2 Hours

Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any **five** questions from the following: 1×5 = 5

- (a) State the type of singularity of the function $f(z) = \frac{1}{\sqrt{z}}$.
- (b) Give an example of a analytic function which is analytic all over the space including infinity.
- (c) What do you mean by singular matrix?
- (d) What is the Fourier transform of a δ -function?
- (e) State scaling property of Fourier transformation.
- (f) Find the inverse of the following matrix:

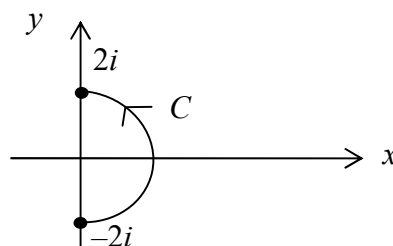
$$\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

- (g) Give an example of a Hermitian matrix.
- (h) Evaluate $\oint_C \frac{e^{-z}}{z+1} dz$, where C is a circle $|z| = \frac{1}{2}$.

GROUP-B

Answer any **three** questions from the following 5×3 = 15

- 2. State and prove Cauchy's integral theorem. 2+3
- 3. (a) Find the square roots of $1 - \sqrt{3}i$. 2
- (b) Evaluate the integral $I = \int_C \bar{z} dz$, where C is the right half of the circle $|z| = 2$ as shown in the figure below. 3



4. Derive the following Taylor series representation. 5

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}}, \quad (|z-i| < \sqrt{2})$$

5. Define Hermitian and Unitary matrix with proper example. Show that the eigen values of a Hermitian matrix are real. 1 $\frac{1}{2}$ + 1 $\frac{1}{2}$ + 2

6. (a) Define Fourier sine and cosine transform. 2

- (b) Find the Fourier transform of the function $f(x)$ defined as: 3

$$f(x) = \begin{cases} \frac{1}{\varepsilon}, & |x| \leq \varepsilon \\ 0, & |x| > \varepsilon \end{cases}$$

GROUP-C

Answer any *two* questions from the following

10×2 = 20

7. (a) Using the method of complex variable, show that 5

$$\int_{-\infty}^{\infty} \frac{\sin x}{x(x^2 - 2x + 2)} dx = \frac{\pi}{2} [1 + e^{-1}(\sin 1 - \cos 1)]$$

- (b) Verify that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane. 2

- (c) Find the harmonic conjugate function of u . 3

8. (a) Expand the function $f(z) = \frac{1}{(z-1)(z-2)}$ between the annular region $|z|=1$ and $|z|=2$. 4

- (b) Diagonalise the following matrix 6

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

9. (a) Consider the following matrix 1+3+3

$$A(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(i) Is the matrix $A(\theta)$ unitary and orthogonal? Justify your answer.

(ii) What are the eigen-values of $A(\theta)$?

- (b) Express $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ as a sum of a symmetric and an antisymmetric matrix. 3

- 10.(a) Discuss the limit at which Fourier series leads to Fourier transformation. 4

- (b) Using Parseval's identity, prove $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$. 4

- (c) Find the Fourier transform of $e^{-a|x|}$, where $a > 0$. 2

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